

## CAD Procedures for Planar Re-Entrant Type Couplers and Three-Line Baluns

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### Abstract

A generalized network model for coupled multiconductor transmission lines is derived based on normal mode parameters. The model is used in the design of planar re-entrant type couplers. By using this network model, the design of a planar re-entrant coupler can be derived from the design of a traditional two-line coupler. The design procedure for single-layer planar baluns using three-conductor transmission lines has also been developed based on this network model. Design procedures and examples are included.

### I. Introduction

It is well known that high coupling is difficult to be achieved by using single layer microstrip coupled lines (due to the need for a small gap between two lines). One of the possible solution is using multilayer circuits with broadside coupling. Design rules for multilayer two-line couplers have been reported earlier [1]. In the section III of this paper, planar re-entrant type couplers are investigated. The re-entrant type coupler using coaxial lines was first described by Cohn [2]. It has the advantages of providing tight coupling without tight tolerance requirements in circuit fabrications. Similar structure was also realized using strip lines by Lavendol and Taub [3]. Re-entrant type couplers using multi-layer microstrip lines (shown in Figure 2) are also reported [4-6]. However, the available design rules are still based on the design concepts for coaxial re-entrant type couplers. It should be noted that there are three different propagation modes for a planar re-entrant type coupler instead of two modes (so called even and odd modes) in a coaxial re-entrant type coupler. Roles of these three modes in producing high coupling can be explained by using network model derived in section II. The network model clearly shows relations between the coaxial and planar re-entrant type couplers. Design procedures and an example are included.

In section IV, a balun configuration using three-coupled lines is investigated. The balun is realized by using two identical 6-dB couplers with different connections and terminations. The final circuit is then optimized by using the three-coupled-line network model. A design example and results are also included.

### II. Generalized Network Model for N-Conductor Coupled Lines

Network models for coupled *inhomogeneous* transmission lines have been reported earlier [7-9]. These generalized network models have been successfully used in the transient analysis of coupled transmission lines. However, design of multiconductor transmission-line circuits based on these models has not been reported. In this section, a generalized network model suitable for this purpose and based on normal mode parameters is described. The application of this model in the design of planar Marchand baluns has been reported earlier [1]. The design of a planar re-entrant type coupler and a three-line balun based on this model will be discussed in the following two sections.

The impedance matrix  $\mathbf{Z}$  for a section of coupled transmission lines (with  $N$  conductors) can be written in congruent form as:

$$\mathbf{Z} = \mathbf{X} \mathbf{M} \mathbf{X}^T$$

$$= \begin{bmatrix} \mathbf{R} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} \end{bmatrix} \begin{bmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{D} & \mathbf{D}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{1} & \mathbf{-1} \\ -\mathbf{D} & \mathbf{D}^{-1} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{P} \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} \end{bmatrix}^T \quad (1)$$

where  $\mathbf{M}$  is the product of the three inner matrices in (1),  $\mathbf{R}$  is the voltage eigenvector matrix ( $R_{ij}$  means the voltage at conductor  $i$  for mode  $j$ , when voltage at conductor 1 is one volt), and

$$\mathbf{D} = \begin{bmatrix} e^{-j\theta_1} & e^{-j\theta_2} & \dots & e^{-j\theta_N} \end{bmatrix}_{diag} \quad (2)$$

$$\mathbf{P} = \begin{bmatrix} \frac{Z_{C11}f_{11}}{|\mathbf{R}|} & \frac{Z_{C12}f_{12}}{|\mathbf{R}|} & \dots & \frac{Z_{CIN}f_{IN}}{|\mathbf{R}|} \end{bmatrix}_{diag} \quad (3)$$

where  $\theta_i$  is the electrical length of the transmission line section for mode  $i$ ,  $Z_{Cij}$  is the characteristic impedance of conductor  $i$  for mode  $j$  and  $f_{ij}$  is the cofactor of  $R_{ij}$ . The matrix  $\mathbf{M}$  can be modeled using  $N$  decoupled transmission line sections with different electrical lengths and impedances, which correspond to  $N$  different modes. The congruent transformation of  $\mathbf{M}$  to  $\mathbf{Z}$  by  $\mathbf{X}$  (as indicated in equation (1)) is equivalent to connections of these  $N$  decoupled transmission lines with transformer banks (determined by  $\mathbf{X}$ ) on both sides. This results in a network model as shown in Figure 1.

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### III. Planar Re-Entrant Type Coupler Design

By using the network model derived in section II, the planar re-entrant type coupler (Figure 2) is modeled as shown in Figure 3. The transmission line section A in the equivalent circuit corresponds to the “odd” mode, for which the voltages at the two top conductors are out of phase ( $R_{21}=-1$ ) and the middle conductor works like a virtual ground ( $R_{31}=0$ ). The transmission line section B and C correspond to “even” modes. For these two “even” modes, the voltages at the two top conductors are equal ( $R_{22}=R_{23}=1$ ). Physically, the three conductors retain almost equal potential in one of these two “even” modes (say mode B). This means  $R_{32} \approx 1$ . However, in the other “even” mode (mode C), the middle conductor has much lower potential ( $R_{33}$  is much smaller than one). This equivalent circuit for planar re-entrant type couplers can be compared with the equivalent circuit for symmetrical two coupled lines in Figure 4 (derived from Figure 1 with  $N=2$ ,  $R_{21}=-1$ ,  $R_{22}=1$ ,  $R_{C11}=Z_{00}$  and  $R_{C12}=Z_{oe}$ ). Thus, it may be inferred that the two “even” modes of the three coupled lines in the re-entrant type coupler put together correspond to the even mode of a symmetrical two coupled lines. Based on this study, the following design procedure has been derived.

#### Design Procedure:

**Step 1.** Calculate the even and odd mode impedances ( $Z_{00}$  and  $Z_{oe}$ ) for a corresponding symmetrical two-line coupler, for the specified coupling coefficient and input impedance.

**Step 2.** The “odd” mode impedance of the planar re-entrant type coupler,  $Z_{C11}$ , is given by  $Z_{00}$  found in Step 1 and is realized using top dielectric layer (see Figure 2) because the middle conductor is like a virtual

ground for this mode. The gap  $s$  between the two top conductors is assumed to be wide enough so that there is no “direct coupling” between them. The line width of top conductors,  $w_1$ , is then determined by calculating the line width for a single layer microstrip line with characteristic impedance  $\tilde{Z}_{00}$  (using dielectric constant and thickness of the top dielectric layer).

**Step 3.** The mode C impedance  $Z_{C13}$  is equal to  $Z_{C11}$  approximately because the low potential of the middle conductor in this mode. The impedance of transmission line section B in Figure 3 is then given by  $(Z_{oe}-Z_{C11})/2$ . Since the three conductors retain about the same potential in this mode, the three conductors can be treated as a single conductor. So, the line width of middle conductor,  $w_2$ , is then determined by calculating the line width for a single layer microstrip line with impedance  $(Z_{oe}-Z_{C11})/2$  (using dielectric constant and thickness of the lower dielectric layer).

**Step 4.** The circuit geometry given by Steps 2 and 3 is analyzed and the exact normal mode parameters ( $Z_{C11}$ ,  $Z_{C12}$ ,  $Z_{C13}$ ,  $R_{32}$  and  $R_{33}$ ) for this planar re-entrant type coupler are calculated. Using the network model in Figure 3 and 4, an equivalent two-line coupler of this planar re-entrant type coupler is found. It has an odd mode impedance  $Z_{00}$  given by  $Z_{C11}$  and even mode impedance  $Z_{oe}$  given by  $(Z_{C13}R_{32}-Z_{C12}R_{33})/(R_{32}-R_{33})$ . The coupling and input impedance of this equivalent two-line coupler are then calculated using well known formulas.

**Step 5.** The results in step 4 are compared with the given specifications. Optimization is carried out by repeating Steps 2 to 4 until the specifications are met.

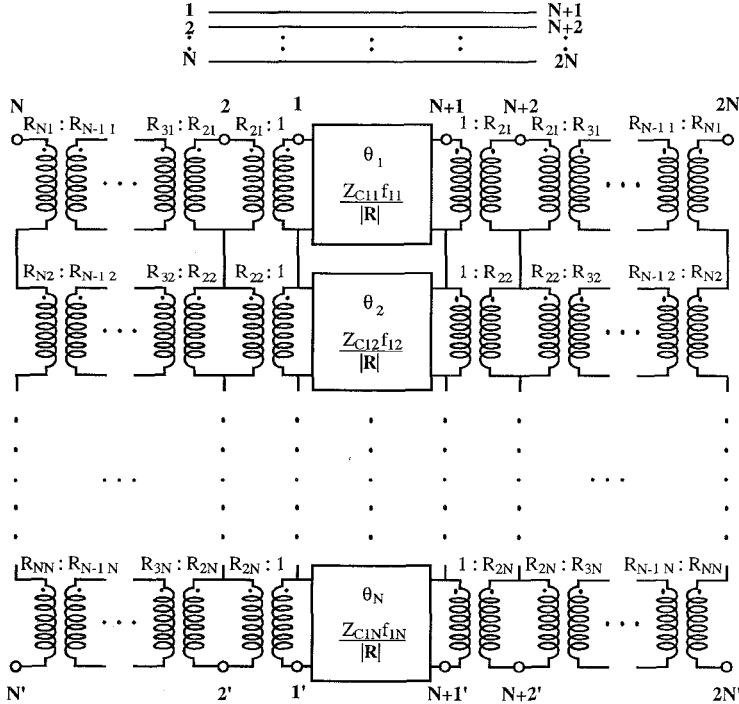


Figure 1. Generalized network model for a section of  $N$ -conductor transmission lines.

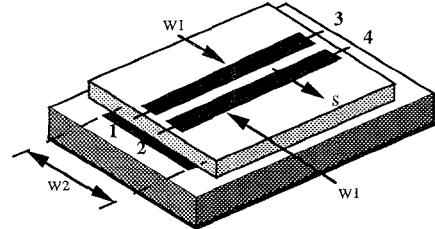


Figure 2. Planar re-entrant type coupler.

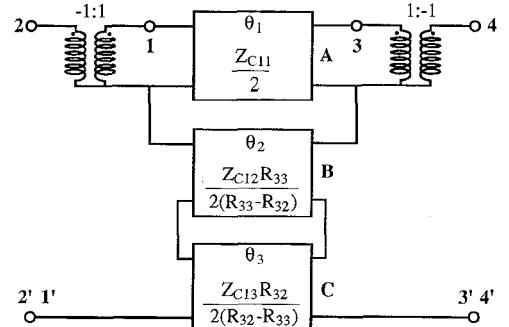


Figure 3. Network model for planar re-entrant type coupler.

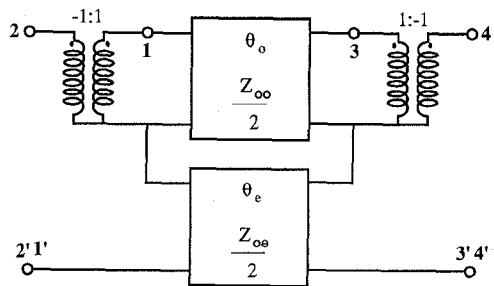


Figure 4. Equivalent circuit for symmetrical two coupled lines.

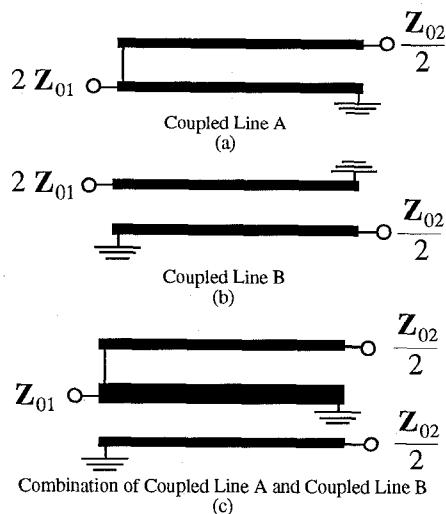


Figure 6. Three-line balun.

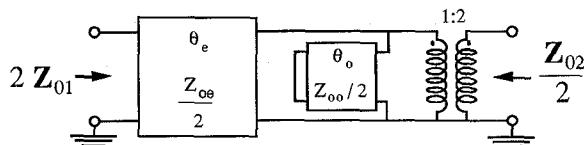


Figure 7. Network model for coupled line A.

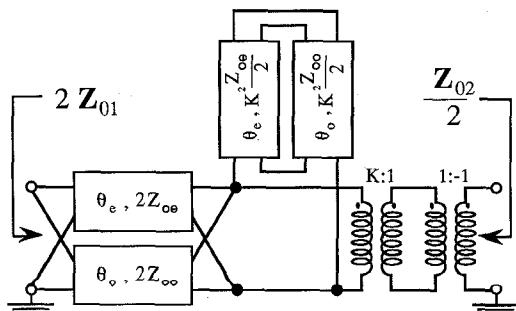


Figure 9. Network model for coupled line B.

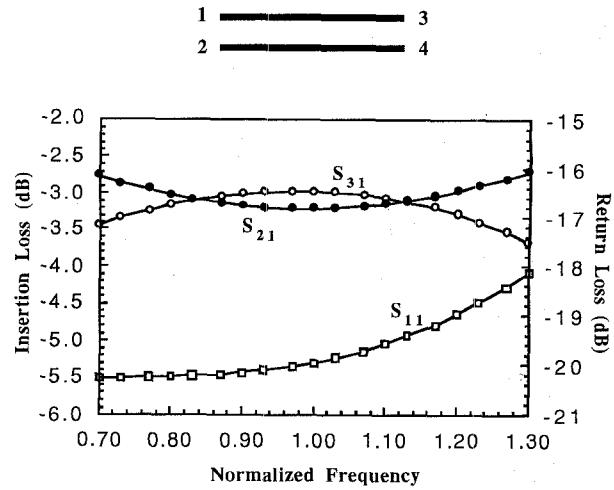


Figure 5. Analysis results for the planar re-entrant type coupler.

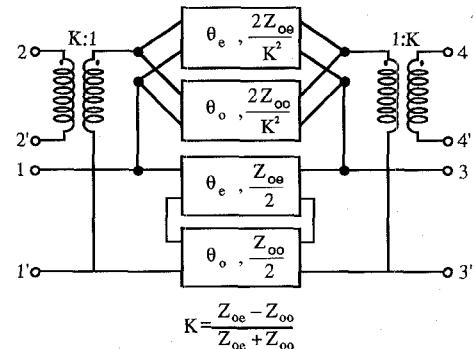


Figure 8. Equivalent network for symmetrical two coupled lines.

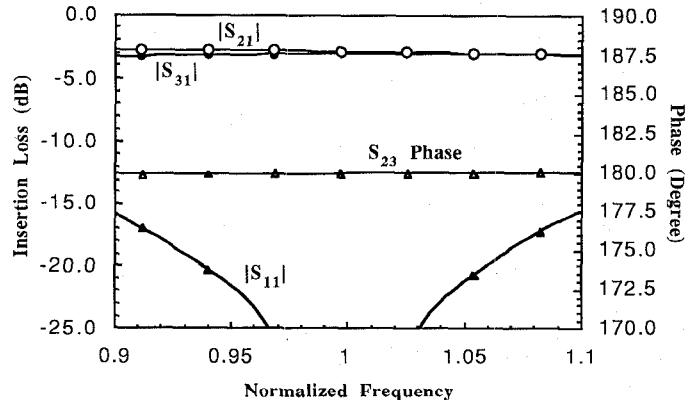
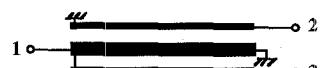


Figure 10. Analysis results for the three-line balun.

As an example, a 3-dB planar re-entrant type coupler with 50 ohm input impedances is designed using alumina substrate of thickness 0.38 mm with a polyamide film of thickness 7.5  $\mu\text{m}$ . For a 50 ohm two-line symmetrical 3-dB coupler, the  $Z_{\text{oo}}$  and  $Z_{\text{oe}}$  are 20.7 ohm and 120.7 ohm respectively. Therefore, the planar re-entrant type coupler has  $Z_{\text{C11}}$  and  $Z_{\text{C13}}$  approximately equal to 20.7 ohm. The  $Z_{\text{C11}}$  and  $Z_{\text{C13}}$  are realized using the polyamide layer and this gives  $w_1$  as 55  $\mu\text{m}$ . The transmission line section **B** is realized using the alumina dielectric layer and this gives  $w_2$  as 225  $\mu\text{m}$ . The gap  $s$  is not a sensitive design parameter and is chosen to be 90  $\mu\text{m}$  so that two top conductors have no “direct coupling” and are “shielded” from ground plane by the middle conductor. This circuit geometry is then analyzed and the results are

$$\begin{aligned} Z_{\text{C11}} &= 21.48 \text{ ohm,} \\ Z_{\text{C12}} &= -431.22 \text{ ohm,} \\ Z_{\text{C13}} &= 27.03 \text{ ohm,} \\ R_{32} &= 1.08, \text{ and} \\ R_{33} &= 0.2 \end{aligned}$$

This re-entrant type coupler is approximately equivalent to a symmetrical two-line coupler with

$$\begin{aligned} Z_{\text{oo}} &= Z_{\text{C11}} = 21.48 \text{ ohm and} \\ Z_{\text{oe}} &= Z_{\text{C12}} R_{33} / (R_{33} - R_{32}) + Z_{\text{C13}} R_{32} / (R_{32} - R_{33}) \\ &= 131.18 \text{ ohm.} \end{aligned}$$

These even and odd mode impedances give 2.87 dB coupling ( $= 20 \log ((Z_{\text{oe}} - Z_{\text{oo}}) / (Z_{\text{oe}} + Z_{\text{oo}}))$ ) and 53.0 ohm input impedance ( $= \sqrt{Z_{\text{oo}} Z_{\text{oe}}}$ ). The analysis results are shown in Figure 5. Further optimization is thus not needed in this case.

#### IV. Three-Line Balun Design

The three-line balun (Figure 6.(c)) discussed in this section is a combination of two symmetrical coupled lines **A** and **B** with different terminations as shown in Figures 6.(a) and 6.(b). Coupled line **A** is modeled by applying the termination conditions to the network in Figure 4 and the final simplified network is shown in Figure 7. The coupled line **B** is modeled differently in order to get a more clear comparison. The network model for symmetrical two coupled lines shown in Figure 4 can also be reconfigured as a network shown in Figure 8. It should be noted that this model is the same as the network proposed by Malherbe [10] when the phase velocities of the two modes are equal. Therefore, this model (with two different phase velocities incorporated) is a generalization of Malherbe’s model for *inhomogeneous* coupled lines. Using this new model, the coupled line **B** can be modeled and simplified as shown in Figure 9.

For the total circuit to function like a balun, the circuits in Figure 7 and Figure 9 must be exactly the same except the last (1:1) transformer in Figure 9. It can be easily shown that, if the difference in two phase velocities is ignored, the coupled lines **A** and **B** (in Figure 6) should have the same even mode and odd mode impedances. Also, the even mode impedance  $Z_{\text{oe}}$  should be three times of the odd mode impedance  $Z_{\text{oo}}$  (i.e., they are 6-dB couplers if matched at all ports) and  $Z_{\text{oe}}$  should be equal to  $\sqrt{Z_{\text{01}} Z_{\text{02}}}$ .

As an example, a three-line balun ( $Z_{\text{01}}=Z_{\text{02}}=100$ ) is designed using 15 mil alumina substrates. The  $Z_{\text{oe}}$  and  $Z_{\text{oo}}$  are then 100 ohm and 33.3 ohm respectively and are realized by two

microstrip lines with 170  $\mu\text{m}$  line widths and 35  $\mu\text{m}$  separation. The final three-line balun is built by using two of this coupled-line section with corresponding terminations as shown in Figure 6.(c). The analysis results of this balun are shown in Figure 10. It is noted that this kind of balun has about 15% bandwidth.

#### V. Concluding Remarks

A generalized network model for multiconductor transmission lines has been developed. This model has been used to formulate the design procedures for planar re-entrant type couplers and three-line baluns. Examples are given and the procedures are justified by network analysis.

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